Old Dominion University Research Foundation

P. 19

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Recognition of 3-D Symmetric Objects from Range Images in Automated Assembly Tasks

by

Nicolas Alvertos, Principal Investigator

and

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Final Report
For the period ended December 31, 1990

Prepared for
National Aeronautics and Space Administration
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Under
Master Contract NAS-1-18584
Task Authorization No. 88
Plesent W. Goode IV, Technical Monitor
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December 1990

(NASA-CR-187664) RECOGNITION OF 3-D SYMMETRIC OBJECTS FROM RANGE IMAGES IN AUTOMATED ASSEMBLY TASKS Final Report, period ending 31 Dec. 1990 (Old Dominion Univ.) 19 p

N92-26656

Unclas
G3/31 0319211

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Abstract

This research presents a new technique for the three-dimensional recognition of symmetric objects from range images. Beginning from the implicit representation of quadrics, a set of ten coefficients is determined for symmetric objects like spheres, cones, cylinders, ellipsoids and parallelepipeds. Instead of using these ten coefficients trying to fit them to smooth surfaces (patches) based on the traditional way of determining curvatures, a new approach based on two-dimensional geometry is utilized. For each symmetric object a unique set of two-dimensional curves is obtained from the various angles at which the object is intersected with a plane. Utilizing the same ten coefficients obtained earlier and based on the discriminant method, each of these curves is classified as a parabola, a circle, an ellipse, or a hyperbola. Each symmetric object is found to possess a unique set of these two-dimensional curves whereby it can be differentiated from the others. In other words, it is shown that instead of using the three-dimensional discriminant which involves evaluation of the rank of its matrix, it is sufficient to utilize the two-dimensional discriminant which only requires three arithmetic operations. This approach seems to be more accurate and computationally inexpensive compared to the traditional approaches.

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1. Introduction

The rapid development of several active and passive range finding techniques has prompted range depth information to be a much more attractive data for the recognition of three-dimensional objects. Location and description of 3-D objects from natural light images are often difficult to obtain. Range images on the other hand give a more detailed and direct geometric description of the shape of the 3-D object. Several techniques [1, 2] have been proposed to solve problems such as range image segmentation and description of three-dimensional surface patches. Most of these methods rely heavily on differential geometry of smooth surfaces and, in the process, evaluations of surface mean and gaussian curvatures. Though these approaches are long, they still come close to describing objects by means of several smooth surface patches.

In this research we put forward an approach based on two-dimensional analytic geometry to recognize symmetric objects like cones, cylinders, spheres, ellipsoids, and parallelepipeds. After obtaining a set of ten coefficients [3] from the explicit representation of quadrics, a series of 2-D curves are obtained from the intersection of planes at various orientations with the surfaces. Since the objects considered in this research are symmetric objects, it is observed that only two planes are sufficient to generate a unique set of curves distinguishing each object from the other.

2. 3-D Discriminant

In this section we will be summarizing a 3-D approach of classification and reduction of quadrics [4], which also looks into the various invariants of the quadratic form under translation and rotation of 3-D objects.

A general second degree quadric equation can be represented as:

$$F(x,y,z) = ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy + 2px + 2qy + 2rz + d = 0 \quad (1)$$

Define two matrices namely D and Δ such that

$$D = \begin{bmatrix} a & h & g & p \\ h & b & f & q \\ g & f & c & r \\ p & q & r & d \end{bmatrix}$$

and

$$\Delta = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$$

Also let A, B, C, D, E, F, G, H, P, Q, and R denote the cofactors of a, b, c, d, f, g, h, p, q, and r respectively in the determinant Δ and α, β, γ, ϕ, ζ, δ denote the cofactors of a, b, c, f, g, h respectively in the determinant D.

It has been shown in [4] that I, J, D, and Δ, where $I = a + b + c$, $J = \alpha + \beta + \gamma$, D, and Δ the determinants, are invariant for any general coordinate transformation. Combining these four invariants a further set of three absolute invariants, $I \frac{J}{D}$, $\frac{I^2}{J}$, and $I \frac{D}{\Delta}$ is also obtained.

Furthermore, based upon the ranks of the matrices Δ and D, the following classification has been deduced.

Rank	4	3	2	1
3	Central quadric	Cone		
2	Paraboloid	elliptic or hyperbolic cylinder	plane-pair intersecting in a line	
1		Parabolic cylinder	parallel plane-pair	Coincident plane-pair

3. Surface Characterization

We will consider the representation and intersections (by two planes) of five symmetric objects : the ellipsoid, the cylinder, the sphere, the cone and the parallelepiped. Assume that all the objects are resting on planes parallel to the yz plane and that their axis of rotation is parallel to any one axis of the coordinate axis. Then in relation to equation (1) the product terms, namely yz, xz, and xy will be missing. We will refer to plane 1 as the plane that intersects the object parallel to the yz-axis, i.e., x = constant. Also let us refer to plane 2 as the plane that intersects the object parallel to the xz-axis, i.e., y = constant.

Consider the equation of an ellipsoid (spheroid) resting on a plane parallel to yz plane and its axis of revolution parallel to y-axis :

Equation (1) reduces to the form

$$F(x,y,z) = ax^2 + by^2 + az^2 + 2px + 2qy + 2rz + d = 0 \tag{2}$$

which further reduces to

$$\frac{\left[x + \frac{p}{a}\right]^2}{\frac{1}{a}} + \frac{\left[y + \frac{q}{b}\right]^2}{\frac{1}{b}} + \frac{\left[z + \frac{r}{a}\right]^2}{\frac{1}{a}} - 1 = 0 \tag{3}$$

only if $d = \frac{p^2}{a} + \frac{q^2}{b} + \frac{r^2}{a} - 1$ and also $a > 0$, $b > 0$.

Consider the intersection of the ellipsoid with plane 1, i.e., $x = k$, where $\frac{-p}{a} - \sqrt{\frac{1}{a}} < k < \frac{-p}{a} + \sqrt{\frac{1}{a}}$, then,

$$\frac{(y + \frac{q}{b})^2}{\frac{1}{b} - \frac{(ak + p)^2}{ab}} + \frac{(z + \frac{r}{a})^2}{\frac{1}{a} - \left[\frac{ak + p}{a}\right]^2} - 1 = 0 \tag{4}$$

which is the equation of an ellipse.

Let's now consider the intersection of the ellipsoid with plane 2, i.e., y=k, where $\frac{-q}{b} - \sqrt{\frac{1}{b}} < k < \frac{-q}{b} + \sqrt{\frac{1}{b}}$, then,

$$\frac{(x + \frac{p}{a})^2}{\frac{1}{a} - \frac{(bk + q)^2}{ab}} + \frac{(z + \frac{r}{a})^2}{\frac{1}{a} - \frac{(bk + q)^2}{ab}} - 1 = 0 \tag{5}$$

which is the equation of a circle.

Next let's consider the general representation of a cylinder resting on a plane parallel to yz plane and its axis of revolution parallel to x-axis. Equation (1) then reduces to

$$F(x,y,z) = by^2 + bz^2 + 2qy + 2rz + d = 0 \tag{6}$$

which is the same as

$$F(x,y,z) = \frac{\left[y + \frac{q}{b}\right]^2}{\frac{1}{b}} + \frac{\left[z + \frac{r}{b}\right]^2}{\frac{1}{b}} - 1 = 0 \tag{7}$$

only if $d = \frac{q^2}{b} + \frac{r^2}{b} - 1$ and also $b > 0$.

Intersection of the cylinder with the plane 1 would no longer affect its representation, since it is independent of the variable x. Hence the resultant curve intercepted is the same as represented by equation (7), which is a equation of a circle.

Consider the case where the cylinder is intersected with plane 2, i.e., y=k, where $\frac{-q}{b} - \sqrt{\frac{1}{b}} < k < \frac{-q}{b} + \sqrt{\frac{1}{b}}$. Then,

$$\left[z + \frac{r}{b}\right]^2 = \frac{1}{b} - \left[\frac{bk + q}{b}\right]^2 \tag{8}$$

Solving for z generates the equation of a pair of parallel lines. In the case where the cylinder is rotated in space, the representation of the cylinder will be of the form of equation (1). Proceeding as usual and intersecting with the two planes would generate curves which can then be recognized utilizing the approach shown in section 4.

Let us consider the cone next. Again the general second degree equation for the cone, homogeneous in x, y, z, is

$$F(x,y,z) = ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy + 2px + 2qy + 2rz + d = 0 \tag{9}$$

However for a circular cone resting on a plane parallel to the yz plane and its axis of revolution parallel to the x-axis, the representation is

$$F(x,y,z) = ax^2 + by^2 + bz^2 + 2px + 2qy + 2rz + d = 0 \tag{10}$$

where $ab < 0$, and also $d = \frac{p^2}{a} + \frac{q^2}{b} + \frac{r^2}{b}$.

From equation (10), upon completing squares, we have

$$F(x,y,z) = a\left[x + \frac{p}{a}\right]^2 + b\left[y + \frac{q}{b}\right]^2 + b\left[z + \frac{r}{b}\right]^2 + d - \frac{p^2}{a} - \frac{q^2}{b} - \frac{r^2}{b} = 0 \tag{11}$$

Since $d = \frac{p^2}{a} + \frac{q^2}{b} + \frac{r^2}{b}$, equation (11) becomes :

$$F(x,y,z) = -\frac{\left[x + \frac{p}{a}\right]^2}{\frac{-1}{a}} + \frac{\left[y + \frac{q}{b}\right]^2}{\frac{1}{b}} + \frac{\left[z + \frac{r}{b}\right]^2}{\frac{1}{b}} = 0 \tag{12}$$

If $a < 0$, i.e., $b > 0$, intersection of the cone represented by equation (12) with plane 1, i.e., $x = k$, where $\frac{-p}{a} - \sqrt{\frac{-1}{a}} < k < \frac{-p}{a}$, would generate

$$\frac{\left[y + \frac{q}{b}\right]^2}{\frac{1}{b}} + \frac{\left[z + \frac{r}{b}\right]^2}{\frac{1}{b}} = \frac{\left[k + \frac{p}{a}\right]^2}{-\frac{1}{a}} \tag{13}$$

where $-\frac{1}{a}$ is a positive quantity. The above equation is that of a circle.

Intersection of the cone with plane 2, i.e., $y = k$, where $\frac{-q}{b} - \sqrt{\frac{1}{b}} < k < \frac{-q}{b} + \sqrt{\frac{1}{b}}$, would generate

$$\frac{\left[x + \frac{p}{a}\right]^2}{\frac{-1}{a}} - \frac{\left[z + \frac{r}{b}\right]^2}{\frac{1}{b}} = \frac{\left[k + \frac{q}{b}\right]^2}{\frac{1}{b}} \tag{14}$$

where $-\frac{1}{a}$ is a positive quantity. The above equation is that of a hyperbola.

The sphere is considered next. Equation (1) for a sphere resting on a plane parallel to the yz-plane is of the form:

$$F(x,y,z) = ax^2 + ay^2 + az^2 + 2px + 2qy + 2rz + d = 0 \tag{15}$$

or

$$F(x,y,z) = \frac{\left[x + \frac{p}{a}\right]^2}{\frac{1}{a}} + \frac{\left[y + \frac{q}{a}\right]^2}{\frac{1}{a}} + \frac{\left[z + \frac{r}{a}\right]^2}{\frac{1}{a}} - 1 = 0 \tag{16}$$

only if $d = \frac{p^2}{a} + \frac{q^2}{b} + \frac{r^2}{a} - 1$.

Consider the case when the sphere is intersected with plane 1, i.e., x = k, where $\frac{-p}{a} - \sqrt{\frac{1}{a}} < k < \frac{-p}{a} + \sqrt{\frac{1}{a}}$. Then,

$$\frac{\left[y + \frac{q}{a}\right]^2}{\frac{1}{a} - \left[\frac{ak + p}{a}\right]^2} + \frac{\left[z + \frac{r}{a}\right]^2}{\frac{1}{a} - \left[\frac{ak + p}{a}\right]^2} - 1 = 0 \tag{17}$$

which is the equation of a circle.

A similar equation evolves when the sphere is intersected with plane 2, in which case y = k, where $\frac{-q}{a} - \sqrt{\frac{1}{a}} < k < \frac{-q}{a} + \sqrt{\frac{1}{a}}$, and subsequently equation (16) becomes:

$$\frac{\left[x + \frac{p}{a}\right]^2}{\frac{1}{a} - \left[\frac{ak + q}{a}\right]^2} + \frac{\left[z + \frac{r}{a}\right]^2}{\frac{1}{a} - \left[\frac{ak + q}{a}\right]^2} - 1 = 0 \tag{18}$$

which again represents a circle.

The next object considered is a parallelepiped. Since planar surfaces cannot be represented with quadratic equations, we consider a plane of the parallelepiped.

The general equation of a plane from equation (1) is of the form

$$2px + 2qy + 2rz + d = 0 \tag{19}$$

Intersection with plane 1, i.e., x = k, will generate

$$2qy + 2rz + d + 2pk = 0 \tag{20}$$

which is the equation of a line.

Similarly, intersection of the above plane with plane 2 would generate the line

$$2px + 2rz + d + 2qk = 0 \tag{21}$$

Following up from the ellipsoid to the parallelepiped, we see that a unique set of curves has been obtained for each of the surfaces using just two planes. The table below summarizes the above results.

object : plane	Ellipsoid	Cylinder	Cone	Sphere	Parallelepiped
x=k	ellipse	circle	circle	circle	line
y=k	circle	line	hyperbola	circle	line

4. 2-D Discriminant

Once a unique set of 2-D curves has been obtained for each 3-D surface, the next objective is to recognize each of these 2-D curves based upon the available coefficients.

Given a conic of the form

$$F(x,y) = Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 \tag{22}$$

the measure $B^2 - 4AC$ discriminates the curves as one of the following [5] :

If $B^2 - 4AC < 0$, then the conic is an ellipse or a circle.

If $B^2 - 4AC = 0$, then the conic is a parabola.

If $B^2 - 4AC > 0$, then the conic is a hyperbola.

Although two planes were sufficient enough to distinguish each of the above five objects from the other, a more indepth analysis of intersection of objects with planes at different orientations, evolves a variety of curves for each of the object. A cylinder for example will generate curves like the parabola, ellipse along with circle and lines generated above, when intersected with planes at different angular orientations. A cone generates curves like parabolas, ellipses, circles, hyperbolas and isosceles triangle when intersected with planes at varying orientations. The angular bounds for which a plane intersecting a surface results into the same 2-D curve must also be investigated.

5. Determining the 3-D Coefficients

In this section, we will briefly present the generation of the ten coefficients which describe 3-D surfaces [3]. The second degree quadric equation as before is represented by

$$F(x,y,z) = ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy + 2px + 2qy + 2rz + d = 0 \tag{23}$$

Our aim is to obtain a surface which best fits the given set of range data. This surface will have the least mean squared error measure for the given data points. Hence it remains to be determined which ten coefficients that describe a quadric surface so as to render the least mean squared error measure. Equation (23) in vector notation will become

$$F(x,y,z) = \mathbf{a}^T\mathbf{p} = 0 \tag{24}$$

where $\mathbf{a}^T = [\ a\ b\ c\ 2f\ 2g\ 2h\ 2p\ 2q\ 2r\ d\ ]$ and $\mathbf{p} = [\ x^2\ y^2\ z^2\ yz\ zx\ xy\ x\ y\ z\ 1\ ]$.

The error measure for any data point (x, y, z) can be measured by substituting in F(x, y, z). If this point lies exactly on the surface then, F(x, y, z) = 0 , meaning that the error is zero.

The mean squared error then is given as

$$E = \min_{\mathbf{a}} \sum_{S} ||F||_2 \tag{25}$$

In vector notation, we have

$$E = \min_{\mathbf{a}} \sum_{S} \mathbf{a}^T \mathbf{p}\mathbf{p}^T \mathbf{a} = \min_{\mathbf{a}} \mathbf{a}^T R \mathbf{a} \qquad (26)$$

where R is the scatter matrix for the data set equal to

$$R = \sum_{S} \mathbf{p}\mathbf{p}^T \qquad (27)$$

Minimizing E would lead to a trivial solution of **a** and hence using Lagrange's method, a new function

$$U = G(\mathbf{a}) - \lambda K(\mathbf{a}), \qquad (28)$$

with an undetermined constant λ and a constraint K evolves. Solving $\frac{\partial u}{\partial \mathbf{a}}$ and $K(\mathbf{a}) = k$ simultaneously, we find **a** and λ to give a minimum solution. Groshong and Bilbro [3] give a detailed analysis for the derivation of the constraint matrix K. Using their results directly gives,

$$K = \begin{bmatrix} K_2 & 0 \\ 0 & 0 \end{bmatrix} \qquad (29)$$

where the constraint matrix

$$K_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/2 \end{bmatrix} \qquad (30)$$

Further as in [3] the coefficient vector $\mathbf{a}_i$ corresponding to the least eigenvalue λ_i for the minimum error solution for the given data is obtained.

6. Experimental Results and Conclusions

For experimental purposes, range image data of a sphere and a cylinder were considered. The raw data of both the sphere and cylinder images were noisy (salt and pepper noise). Median filters of mask sizes 3 x 3 and 5 x 5 were used to remove the noise and the effect of these filters on the range data was studied. Figure 1 shows the raw image of the sphere. Figure 2 is the 5 x 5 filtered image of figure 1. Similarly figures 3 and 4 are the corresponding images for a cylinder. For each set of the raw data and the processed images, the ten coefficients were obtained.

Each of the processed images of both the sphere and the cylinder were intersected with the two planes (one parallel to yz-plane, the other parallel to xz-plane). The results obtained for the sphere are tabulated below.

Sphere Images	Raw	3 x 3 filt.	5 x 5 filt.
plane 1, x = k	Ellipse	Ellipse	Circle
plane 2, y = k	Ellipse	Ellipse	Circle

A decision on the curve being an ellipse or a circle was made based upon parity and the disparity of the x^2, y^2, and the z^2 coefficients.

Experiments conducted with the raw and the processed images of the cylinder led to the following results.

Cylinder Images	Raw	3 x 3 filt.	5 x 5 filt.
plane 1, x = k	Ellipse	Line or Ellipse	Line
plane 2, y = k	Ellipse	Ellipse	Ellipse

As seen form the tabulated results, the raw images come close in generating the desired curves for each of the objects, but at the same time a 5 x 5 filter in either case generates the exact 2-D curves.

In this research we have put forward a new technique to recognize symmetric three dimensional range objects. The intersection of a set of planes with each of the objects results in a unique set of two-dimensional curves, which is sufficient enough to discriminate each from the other. In the future research, work will be concentrated on recognizing a bigger set of three dimensional objects like the paraboloids, the hyperboloids of one and two sheet and several other useful 3-D objects. Work will also be directed towards the determination of the bounds through which each object yields a unique set of curves.

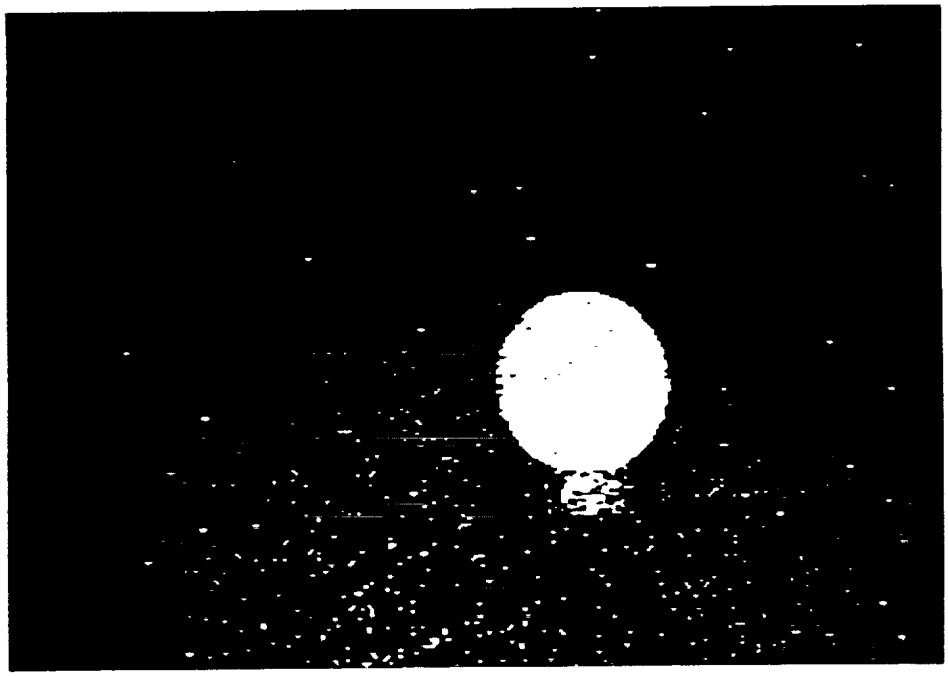

Figure 1. Raw image of the sphere.

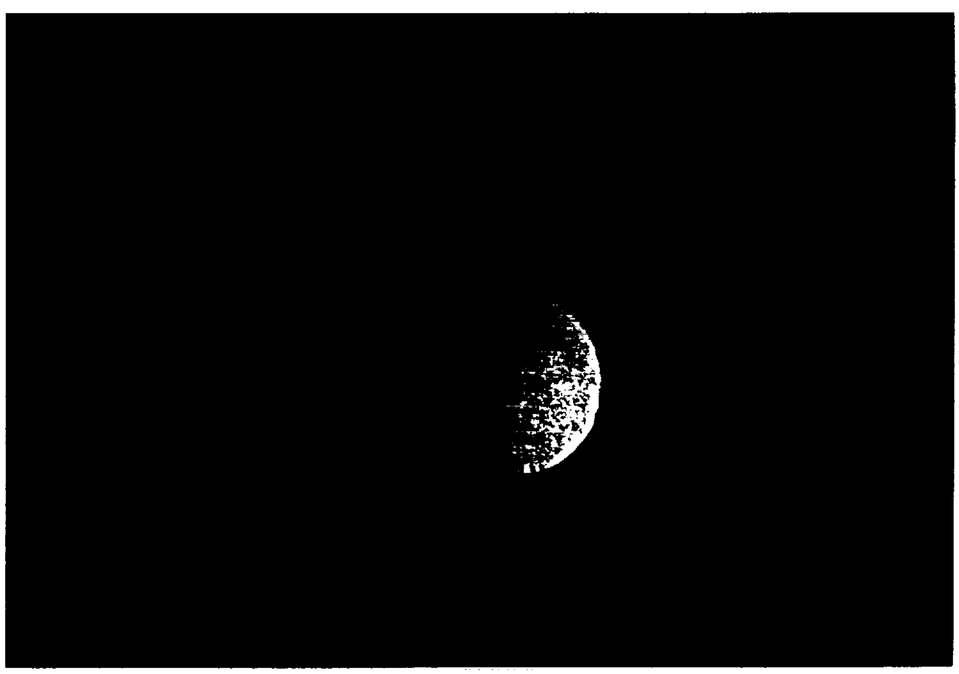

Figure 2. 5 x 5 median filtered image of the raw sphere.

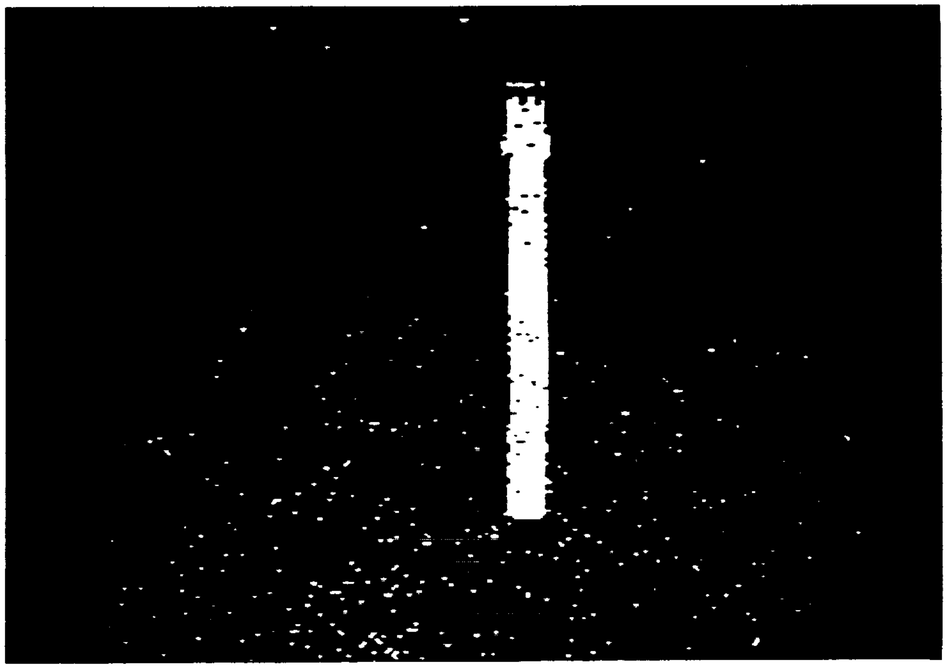

Figure 3. Raw image of the cylinder.

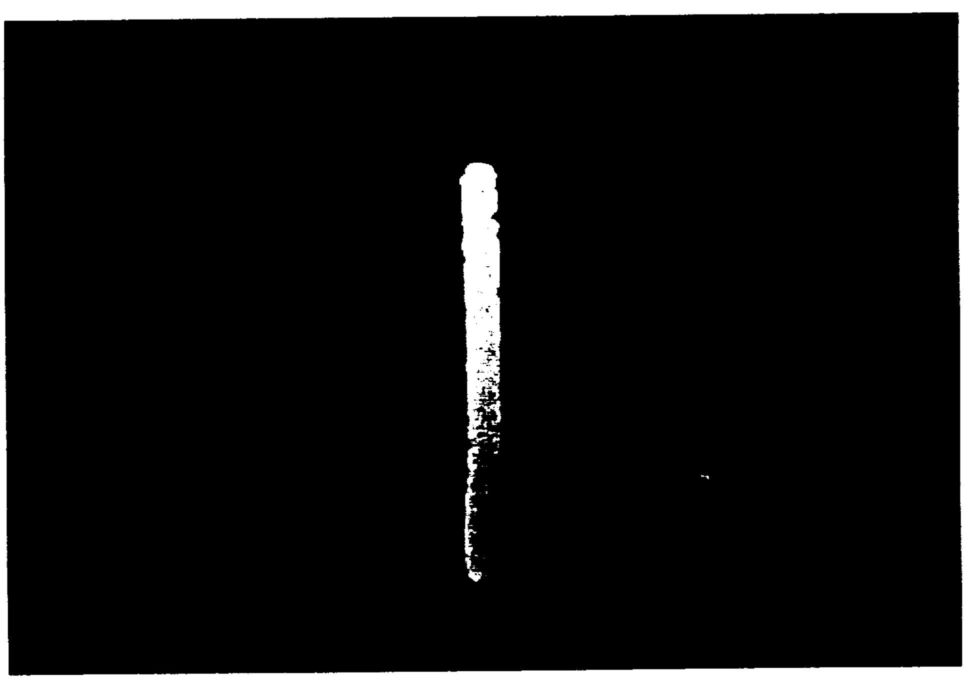

Figure 4. 5 x 5 median filtered image of the raw cylinder.

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